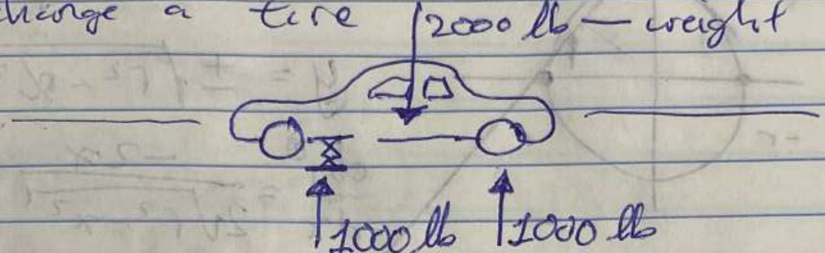
6.5 Work and Fluid Forces

- work done by a constant force

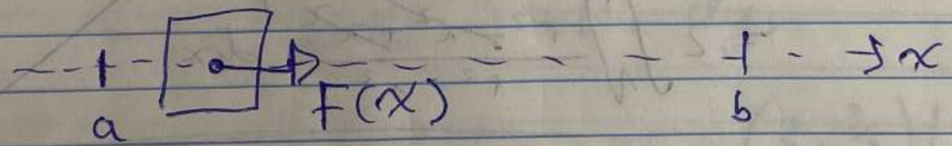
$$W = Fd \qquad \text{units } N{\cdot}m \text{ in SI}$$

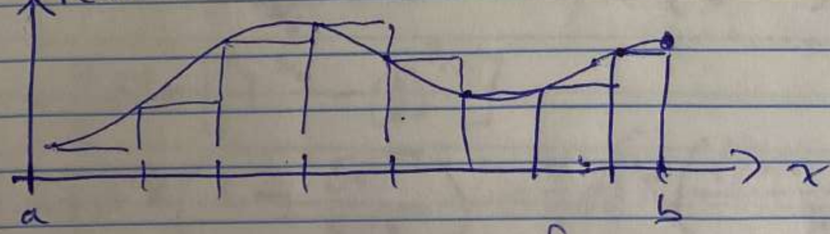
__example.__ jack up a 2000-lb car to change a tire



Jack it up 1.25 ft in the air
the jack has ~~d~~ applied a constant force 1000 lb through this process
and has done $1000\text{ lb} \cdot 1.25\text{ ft} = 1250\text{ ft-lb}$ of work

- work done by a variable force on a line





incremental work of subprocesses

$$\Delta W = F \Delta x$$

work is the integral over distance

$$W = \int_a^b \underbrace{F(x)}_{N}\,\underbrace{dx}_{m}$$

if F is constant. it collapses back to the previous formula $W = \int_a^b F\,dx = (F)(b-a)$

Example, let $F(x) = 1/x^2$ N for a pushing process ~~across~~ from $x = 1m$ to $x = 10m$

$$[Nm] \quad W = \int_{1m}^{10m} \left(\frac{1}{x^2}(N)\right)(dx\ (m))$$

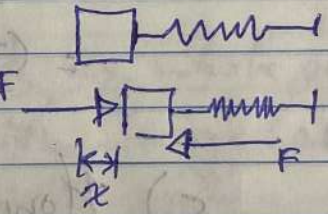
$$= \left[\frac{-1}{x}\right]\Big|_{x=1}^{10}$$

$$= \frac{-1}{10} - \left(-\frac{1}{1}\right)$$

$$= \frac{9}{10} \text{ Nm (J)}$$

Hooke's Law for springs

$$F = kx$$



compression or tension of a spring results in a force proportional to displacement

$x = 0$ is the equilibrium of the spring

example a spring has a natural length of 10 in, and is streched to 14 in by an 800 lb force.

a) find force const.

$$k = F/(x - x_0) = \frac{800\,lb}{14in - 10in}$$

$$k = 200\ lb/in \text{ (pounds per inch)}$$

b)

b) how much work is done to stretch the spring from 10 in to 12 in?

$$F = k(x - x_0) \qquad x_0 = 10\,in \quad k = 200\, lb/in$$

$$W = \int_{10\,in}^{12\,in} k(x - x_0)\,dx \overset{or}{=} \left(\int_0^2 kx\,dx\right)$$

$$= \cancel{\int_{10}} k\left(\frac{(x-x_0)^2}{2}\right)\Big|_{x=10}^{12}$$

$$= k\left[\frac{1}{2}\left(\frac{(12-10)^2}{2} - \frac{(10-10)^2}{2}\right)\right]$$

$$\frac{lb}{in} \qquad in^2$$

$$= \frac{k}{2}\cdot[2) = k = 200\, lb$$

$$= \frac{(200\, lb/in)}{2}(2\, in^2) = 200\, lb\cdot in$$

c) How far beyond the natural length will a 1600 lb force stretch the spring?

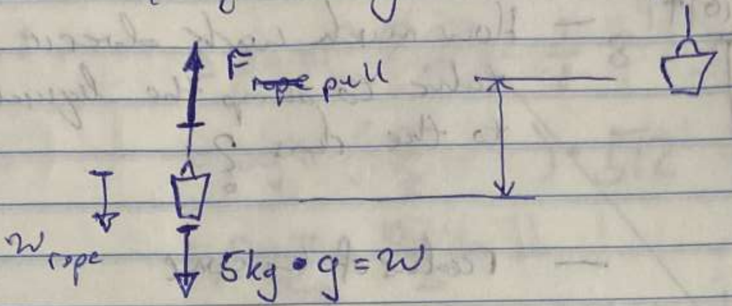
$$F = k(x - x_0)$$

$$x - x_0 = \frac{F}{k} = \frac{(1600\, lb)}{(200\, lb/in)} = 8\, in$$

30

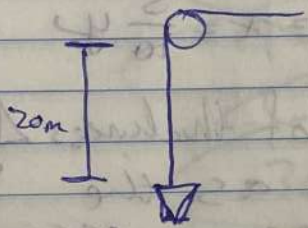
Lifting objects and pumping liquids from containers.

example 5kg bucket lifted on a rope



$g = 9.8 \frac{N}{kg}$

rope weighs $0.08\ kg/m$, starting at 20m



when the rope is lifted by x its length is $(20-x)$ m and therefore its weight changes

weight force through the whole process is

$$w(x) = g \cdot 5\cancel{0} kg + g \cdot (0.08 kg/m)(20-x)(m)$$

$$W = \int_0^{20} \left(g \cdot 5\cancel{0} + g(0.08)(20-x)\right) dx$$

$$= g\left(5\cancel{0}x + 0.08\left(20x - \frac{x^2}{2}\right)\right)\Bigg|_{x=0}^{20}$$

$$= g_{\frac{N}{kg}}\left(5\cancel{0}_{kg} \cdot (20)_{m} + 0.08_{kg/m}\left(20_m \cdot 20_m - \frac{(20)^2}{2}_{m}\right)\right)$$

$$= \left(9.8 \frac{N}{kg}\right)\left(10\cancel{0}\ kgm + 0.08 \frac{kg}{m} \cdot 200 m^2\right)$$

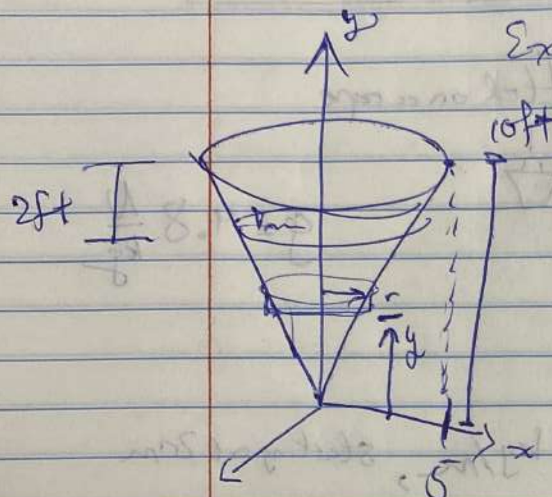
$$= \left(9.8 \frac{N}{kg}\right)\left(100\cancel{0}\ kgm + 16\ kgm\right)$$

close to 1160 J

is 1136.8 J

Pumping Fluids

Example Conical tank with oil $\rho = 57\ lb/ft^3$



- How much work does it take to pump the liquid to the rim?

- radius of the cone

$$y = \frac{10}{5}x$$

$$r(y) = x = \frac{5}{10}y$$

- Break the fluid into slabs of thickness Δy and calculate work as the change in potential energy associated with bringing each slab to the top

Δ gravitational force (weight) $\quad$ Δ distance traveled (vertical)

Δy $\qquad \Delta V = (\pi r(y)^2)\Delta y$

$\longmapsto r(y)$

weight of this guy is $(\rho \Delta V g)$

work for this guy is $(\rho g \Delta V)\cdot(10ft - y)$

$$\Delta W = \rho g (10ft - y)\pi(r(y)^2)\Delta y$$

$$W = \rho g \int_0^8 (10-y)\pi\left(\frac{5}{10}y\right)^2 dy$$

32

$$\rho g \pi \int_0^8 \frac{10y^2 - y^3}{4}\, dy$$

$$w = \rho g \pi \int_0^8 \frac{5}{2}y^2 - \frac{1}{4}y^3 \, dy$$

$$= \rho g \pi \; \frac{10}{3\cdot 2} y^3 - \frac{1}{4\cdot 4} y^4 \Big|_{y=0}^{8}$$

$$= \rho g \pi \left(\frac{10}{3}(8^3) - \frac{1}{8} 8^4 \right)$$

$$= \rho g \pi \left(\frac{10}{3} - 1 \right) \cdot 512$$ junk!

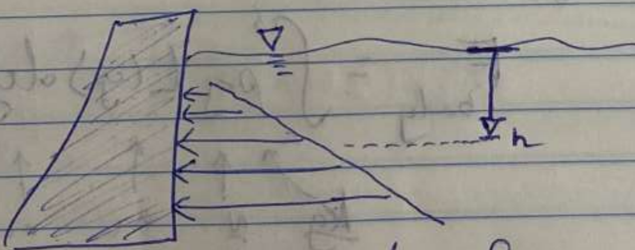
$$= \rho g \pi \frac{1024}{3}$$

$$= (57\, lb/ft^3)\left(\frac{1024}{3} ft^4\right)$$

$$= 19456 \text{ ft-lb}$$

30,561 ft-lb

• Pressure and depth



h : depth below free surface, ρ : mass density

w : fluid weight density,

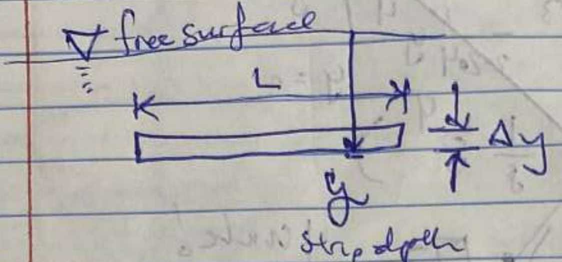
$$P = wh = \rho g h$$

$$[P] = N/m^2 \text{ or } lb/ft^2 \text{ or } lb/in^2 = psi$$

Pascal (Pa)

atmosphere $P_{atm} = 101.3$ kPa

Fluid force ~~at~~ on a constant depth surface



bottom pressure ↑ $F_{bot} = \rho g \cdot y \cdot L$

top pressure ↓ $\rho g \cdot (y - \Delta y) L$

$F_{top} =$

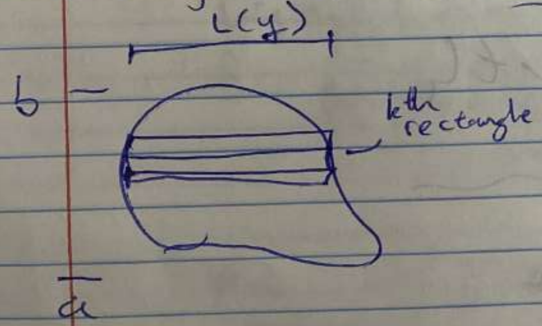
force differential: $\Delta F = F$

bouyant force: $\Delta F = F_{bot} - F_{top}$

$$= \rho g y L - \rho g (y - \Delta y) L$$

$$\Delta F = \rho g L \Delta y$$

- for a body

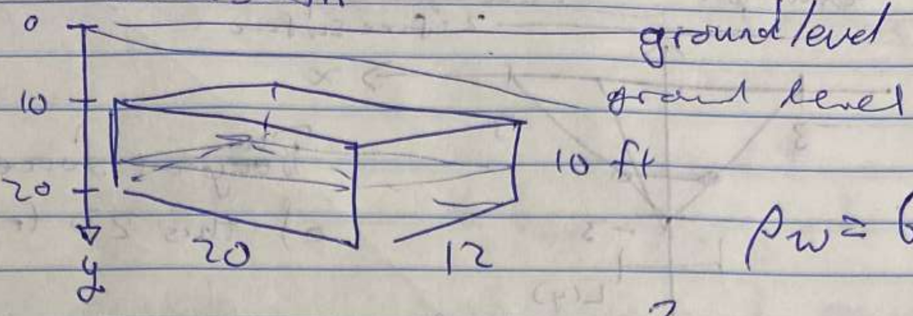


$$\Delta F_k = \rho g (L(y)) \Delta y$$

$$F_{body} = \int_a^b \rho g L(y) dy$$

$\frac{kg}{m^2}$ $\frac{N}{kg}$ m   m

16) water in a cistern



$\rho_w = 62.26 \frac{lb}{ft^3}$

a) how much work to empty?

csec area $A(y) = 240\ ft^2$

$dV = A(y)dy$

integrate in y from 10 to 20

work to remove each slice

$dW = (F)(dist)$

$= (\rho_w dV)(y)$

work to remove all slices

$$W = \int_{10}^{20} \rho_w y (A(y)dy)$$

$$= \rho_w (240 ft^2) \left[\frac{y^2}{2}\right]\Bigg|_{y=10}^{20}$$

remember upload syllabus